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Further Results on the Probability Theory of Capacity Value of Additional Generation

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Abstract—New theoretical results regarding the capacity value of additional generation are presented, the motivation being explanation of results from applied renewables integration studies. Of particular note are the dependence of calculated values on underlying risk level for any capacity of additional generation, the upper limit on capacity value where there is a given probability of near-zero available capacity, and a closed form result for the case where the distribution of available existing capacity may be approximated as an exponential function. Examples of how these main results may be applied are presented.

Keywords - power system planning; power system operation; power system reliability; risk analysis; wind energy

I. INTRODUCTION

CONCEPTS of capacity value are widely used to quantify the contribution of renewable generation technologies within generation adequacy assessments. Specific definitions include Effective Load Carrying Capability (ELCC, the extra demand which the additional generation can support without increasing the chosen risk metric), and Equivalent Firm Capacity (EFC, the completely firm capacity which would give the same risk level if it replaced the additional variable generation). These are calculated with respect to adequacy indices such as the Loss of Load Probability (LOLP) at time of annual peak, or the Loss of Load Expectation (LOLE, the sum over periods of LOLP, or equivalently the expected number of periods of shortage in a given time window).

There are many surveys in the literature of capacity value calculation methods, e.g. [1]–[3]. A number of papers have been published recently on analytical calculation approaches which are valid for small additional capacities [4], [5], or for the special case where the distribution of margin of existing capacity over demand has an exponential tail [6], [7]; these analytical approaches are surveyed in [8]. [9] and [10] provide general surveys of adequacy assessment methods. The IEEE PES LOLE Working Group website contains useful presentations on current industrial adequacy assessment practices [11].

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This paper will introduce a number of new analytical results on the capacity value of additional generation, and show how they can be used to interpret the results of practical calculations. First a survey of existing analytical results regarding capacity value calculations is provided (Section II-A). This includes the ‘time-collapsed’ or ‘snapshot’ picture which may be used to formulate theoretical results on Hierarchical Level I (no network restrictions) capacity value calculations in a common manner for both ‘whole-season’ indices such as LOLE and ‘single period’ indices such as annual peak LOLP; this simplifies the mathematical exposition and helps clarify which what features of distributions drive the capacity value results. The survey is a condensed version of that provided in [12], and forms the basis of the new results to follow.

Section II-B presents a number of useful technical results on capacity values, most significantly the formulation in (7) of the capacity value as a function of required margin k . This provides the most convenient means to explore the dependence of capacity values on shifts in risk level; as EFC and ELCC are special cases of this more general formulation, it unifies them mathematically and permits single proofs of results which apply to either.

The main theoretical contributions are found in Section III:

- The result in Section III-A that, given probability p that available additional capacity Y is below a value a , this places a ceiling on the calculated capacity value of Y .
- Generalisation in Section III-B, to any capacity of additional generation, of the result showing how the dependence of capacity value results on shifts of the distribution of margin is determined by whether the distribution of margin of existing capacity over demand is lighter- or heavier-tailed than exponential; previously this has only been proved for small additional capacities.
- An analytical result in Section III-C for the capacity value of additional generation where the distribution of available existing capacity may be approximated, for capacities below maximum demand, by an exponential function. Unlike the previous result quoted in II-A3, this requires no assumption of statistical independence between additional generation and demand; however like other all other results in this paper it requires conventional capacity to be independent of all else.

These rigorous results are all of practical importance in understanding results from capacity value studies, as shown by the example results in Section V using data described in Section IV. Finally conclusions are presented in Section VI.

II. PROBABILITY THEORY: BACKGROUND AND TECHNICAL RESULTS

In this section, we first briefly survey previous analytical results concerning the capacity value of additional generation which may be derived using probability theory, including the ‘snapshot’ picture in which we work and results which will be generalised in the next section. We then present a number of significant new technical results.

A. Background and Previous Results

1) *Snapshot Theory*: We present first a “time-collapsed” or “snapshot” description of the theory, appropriate to the distributions of the variables involved at a given instant of time. This was introduced in [4] and described fully in [12].

Suppose that existing capacity minus demand is represented by a random variable M , with distribution function F_M and density function f_M . The capacity value of additional generation represented by a random variable Y is in an appropriate sense a deterministic capacity which is equivalent to it in terms of an associated risk. Suppose that $Y \geq 0$ with distribution function F_Y and density function f_Y . We denote its mean and variance by μ_Y and σ_Y^2 respectively. Unless Y is a constant, this capacity value is typically somewhat less than the mean of Y . The two most commonly used definitions of the *capacity value* of Y are:

Effective load carrying capacity (ELCC): the solution ν_Y^{ELCC} of

$$\mathbf{P}(M + Y \leq \nu_Y^{\text{ELCC}}) = \mathbf{P}(M \leq 0) = F_M(0), \quad (1)$$

i.e. the amount of further demand which may be added while maintaining the same level of risk.

Equivalent firm capacity (EFC): the solution ν_Y^{EFC} of

$$\mathbf{P}(M + Y \leq 0) = \mathbf{P}(M + \nu_Y^{\text{EFC}} \leq 0) = F_M(-\nu_Y^{\text{EFC}}), \quad (2)$$

i.e. the amount of deterministic capacity ν_Y^{EFC} whose addition would result in the same level of risk as that of the addition of the random capacity Y .

It is important to note that both ν_Y^{ELCC} and ν_Y^{EFC} depend on the distributions of both M and Y .

2) *Small Additional Capacity*: In the case where M and Y are independent, and in which the variation in Y is small in relation to that in M , [4] showed that to a good approximation if M is continuous (and thus has a density $f_M(m)$),

$$\nu_Y^{\text{EFC}} = \nu_Y^{\text{ELCC}} = \mu_Y - \frac{f'_M(0)}{2f_M(0)}\sigma_Y^2, \quad (3)$$

where the error is negligible in relation to σ_Y^2 as the latter becomes small (in relation to the variation in M). This formula may also be applied in the case where Y and M are not independent, by replacing μ_Y and σ_Y by the mean and SD of Y conditional on $M = 0$, i.e. on being in the critical regime where Y is both required and able to mitigate a shortfall.

3) *Exponential Left Tail of M* : A further special case – which forms the basis of the Garver approximation [6], [7] – arises when M and Y are independent and when the distribution function F_M of M may be treated as exponential below some level m_0 , i.e. $F_M(m) = \exp \lambda_M m$ for $m \leq m_0$ for some $\lambda_M > 0$. The distribution of $M + Y$ is then also exponential below the level m_0 , i.e. for $m \leq m_0$,

$$\mathbf{P}(M + Y \leq m) = \mathbf{P}(M + \nu_Y \leq m), \quad (4)$$

where ν_Y is the solution of

$$\mathbf{E} \exp(-\lambda_M Y) = \exp(-\lambda_M \nu_Y). \quad (5)$$

The results of [6], [7], in particular that $\nu_Y^{\text{EFC}} = \nu_Y^{\text{ELCC}} = \mu_Y$, follow immediately from (5), though this most general form was first presented in [12].

4) *Capacity Values Over Extended Periods of Time*: For practical adequacy calculations, it is most common to calculate indices such as Loss of Load Expectation, the sum of LOLPs in individual sub-periods $\{t\}$ within the extended period (e.g. a future peak season) under study. For simplicity consider the EFC ν_Y^{EFC} . (2) is now replaced by

$$\sum_t \mathbf{P}(M_t + Y_t \leq 0) = \sum_t \mathbf{P}(M_t + \nu_Y^{\text{EFC}} \leq 0), \quad (6)$$

i.e. ν_Y^{EFC} is the additional deterministic capacity which would substitute for the randomly varying additional capacity Y if the same *loss of load expectation* is to be maintained.

In general all of the preceding theory remains applicable. In particular the “extended time” theory collapses to the “snapshot” theory if we are able to regard the latter as corresponding to the position at a *randomly chosen* point in time, provided that the distributions of M and of Y conditional on M are defined appropriately.

B. New Technical Results

1) *Effect of Distribution Shifts*: In order to both to study the effect of varying risk levels and to give a mathematically unified treatment of ELCC and EFC, it is convenient, for all k , to define ν_Y^k be the solution of

$$\mathbf{P}(M + Y \leq \nu_Y^k + k) = \mathbf{P}(M \leq k) = F_M(k). \quad (7)$$

Note that, since $Y \geq 0$, it follows from (7) that $\nu_Y^k \geq 0$ for all k . Comparison of (1) and (7) shows that ν_Y^k is the capacity value of Y corresponding to “shifting” the risk level by k ; more precisely it is the ELCC corresponding to the shifted random variable $M - k$. It now follows that $\nu_Y^{\text{ELCC}} = \nu_Y^0$, while, from (2) and (7), $\nu_Y^{\text{EFC}} = \nu_Y^{-c}$ where c is the solution of $\nu_Y^{-c} = c$. Thus ν_Y^{ELCC} and ν_Y^{EFC} are both particular instances of the ‘generalised’ capacity value ν_Y^k .

2) *General results for M and Y independent*: In this case, (7) may be written either as

$$\int_{-\infty}^{\infty} dm f_M(m) F_Y(\nu_Y^k + k - m) = F_M(k). \quad (8)$$

or as

$$\int_0^{\infty} dy f_Y(y) F_M(\nu_Y^k + k - y) = F_M(k). \quad (9)$$

From (8) (and the monotonicity of distribution functions) it follows that if Y is replaced by an additional capacity Y' which is stochastically larger (i.e. $F_{Y'}(y) \leq F_Y(y)$ for all y) and is Y' is also independent of M , then $\nu_{Y'}^k \geq \nu_Y^k$.

Now suppose that the probability represented by the left side of (7) arises from values of M concentrated in its left tail—as is natural since we are considering small risks, and that the distribution function of M is convex in this region—as is again natural. Then, from (9) and Jensen's inequality (see, for example, [13]),

$$F_M(\nu_Y^k + k - \mu_Y) \leq F_M(k), \quad (10)$$

so that necessarily $\nu_Y^k \leq \mu_Y$. Thus under these conditions the capacity value of Y , in particular the ELCC or the EFC is at most the mean μ_Y of Y .

III. NEW RESULTS

A. Limit on Capacity Value of Y Given Probability of Near-Zero Available Capacity

Suppose that Y and M are independent, and that there is some $a \geq 0$ and $p > 0$ such that $\mathbf{P}(Y \leq a) = p$, then, from (7), for all k ,

$$p\mathbf{P}(M + a \leq \nu_Y^k + k) \leq \mathbf{P}(M \leq k),$$

so that

$$\nu_Y^k \leq a - k + F_M^{-1}\left(\frac{1}{p}F_M(k)\right), \quad (11)$$

regardless of how large may be the values of Y above k – in other words, this probability of near-zero output cannot be fully compensated in the calculation by the possibility of very high output. This is particularly relevant in the case of uncertain generation which may, with some probability $p > 0$, fail entirely – in which case we use (11) with $a = 0$.

In the special case of 'exponential M ' given in Section II-A3, the result (11) reduces to the simple form

$$\nu_Y \leq a + \frac{1}{\lambda_M} \ln p. \quad (12)$$

B. Variation of Results with Risk Level

In the general case, when the above 'exponential M ' and 'small Y ' approximations of F_M are not necessarily available, it is important to understand how the capacity value ν_Y^k varies with variation in the risk level k . For the distribution M , define the (left-tail) hazard rate function

$$\lambda_M(m) = \frac{f_M(m)}{F_M(m)} = \frac{d}{dm} \ln F_M(m), \quad (13)$$

and note that, for any m_1, m_2 ,

$$F_M(m_2) = F_M(m_1) \exp \int_{m_1}^{m_2} dm \lambda_M(m).$$

For M and Y independent, (9) may be rewritten as

$$\int_0^\infty dy f_Y(y) \exp \int_k^{\nu_Y^k + k - y} dm \lambda_M(m) = 1. \quad (14)$$

For the exponential case considered earlier, $\lambda_M(m)$ is constant over values of m in the left-tail region of M ($\lambda_M(m) = \lambda_M$ for all $m \leq m_0$) and so, from (14) the earlier result that ν_Y^k is independent of k is obtained once more. For a distribution whose left tail is lighter than exponential (see Page 2 of [14] for the necessary formal definitions), in general, for values of m in the left-tail region of M (i.e. in the neighbourhood of k), $\lambda_M(m)$ is increasing as m decreases; hence in this case, in order for (14) to remain satisfied, the capacity value ν_Y^k must decrease as k decreases (corresponding to a more extreme level of risk). For a distribution whose left tail is heavier than exponential, we obtain the reverse inequality: for values of m in the left-tail region of M , $\lambda_M(m)$ is decreasing as m decreases, and hence in this case we have the result that the capacity value ν_Y^k increases as k decreases.

C. Exponential existing capacity X , independent of all else

Suppose that $M = X - D$ where X is the existing capacity and D is demand. Suppose also that that the random variable X is independent of all else, and that the distribution function F_X of X is such that, for x below the maximum possible value of the demand D , $F_X(x) = ce^{\lambda_X x}$ for some constant λ_X . (Thus, in particular, the left tail of the distribution of X is exponential.) We then have the following result.

1) *Result:* The distributions of both M and $M+Y$ also have exponential left tails below zero margin, each with the same exponential constant λ_X . Further the additional generation Y shifts the distribution of margin by a capacity value

$$\nu_Y = \frac{1}{\lambda_X} \ln \left(\frac{\mathbf{E}[e^{\lambda_X D}]}{\mathbf{E}[e^{\lambda_X (D-Y)}]} \right), \quad (15)$$

i.e. $P(M + Y \leq m) = P(M \leq m - \nu_Y)$ for $m \leq 0$.

2) *Proof:* The result stated in Section III-C1 follows from two applications of the technical result used in Section II-A3, namely that the convolution of a distribution with an exponential left tail and a distribution whose support takes a minimum value results in a distribution which again has an exponential left tail. This result is applied to obtain both the distributions of $M = X - D$ and of $M + Y = X + Y - D$, noting that X is independent of all else.

3) *Remarks:*

- It is clear from (15) that if the installed capacity of Y is large on a scale of $1/\lambda_X$, then the capacity value ν_Y will be driven primarily by the part of the distribution of Y at relatively low output, and the marginal capacity value will be small relative to the mean of Y
- It is common in R+D work on capacity values to examine dependence of the capacity value on the installed capacity by constructing the additional capacity as $Y = y^+ \Omega$, where y^+ is the installed capacity of additional generation and Ω the load factor (a random variable). It follows that the marginal capacity value as y^+ is perturbed is

$$\frac{\partial}{\partial y^+} \nu_Y = \frac{\mathbf{E}[y^+ \Omega e^{\lambda_X (D - y^+ \Omega)}]}{\mathbf{E}[e^{\lambda_X (D - y^+ \Omega)}]}. \quad (16)$$

The right hand side may be interpreted as a mean of Y , weighted by the factor $e^{\lambda_X(D-y^+Y)}$.

- In the case of a hindcast calculation, in which the (suitably rescaled) empirical historic distribution (d_t, y_t) is used as the predictive joint distribution of (D, Y) (so that e.g. the LOLE is given by $N_y^{-1} \sum_t F_X(d_t - y_t)$ where N_y is the number of years of data), then the expression (15) for the capacity value becomes

$$\nu_Y = \frac{1}{\lambda_X} \ln \left(\frac{\sum_t e^{\lambda_X d_t}}{\sum_t e^{\lambda_X (d_t - y_t)}} \right). \quad (17)$$

- As with all special case results on capacity values, the full calculation is not so computationally intensive that the special case is required in order to speed up calculations. However, as is typically the case it is valuable in giving transparent insights into the parts of the distributions of M and Y which drive the results, and also the parametrisation by λ_X of the distribution of X may be useful in investigating the sensitivity of capacity value results to the distribution of X .

IV. DATA FOR EXAMPLES

A. Demand Data

For this paper, coincident Great Britain wind resource and demand data are available for the seven winters 2005-12. The demand data are supplied by National Grid, the GB System operator. This is based on historic metered demand (available publicly at [15]); and an estimate of historic distribution-connected wind output is added back on to the transmission-metered demand series, giving a consistent series of gross demand as seen at grid supply points under an assumption of unchanging underlying demand patterns.

Historic demand data may be rescaled to a required underlying level using each winter's average cold spell (ACS) peak demand. ACS peak demand is the standard measure of underlying peak demand level in Great Britain; conditional on a given underlying demand pattern, it is the median out-turn winter peak demand [16].

The historic demand data is on a half-hourly time resolution. However as for some examples coincident historic wind and demand series are required, the demand series is converted to hourly by taking for each hour the higher of the two half-hourly demands contained therein.

In order to account for the operator's practice of taking emergency measures such as demand reduction in preference to eroding the frequency response which protects the system against sudden losses of infeed, 700 MW is added to all demand values to represent the response which is supplied by conventional generating units. This is consistent with the statutory Capacity Assessment Study [17].

B. Wind Data

An hourly wind power resource dataset has been supplied by National Grid, combining wind speed resource data from NASA's MERRA [18] reanalysis dataset (winters 1979-2012) with installed capacity scenarios based on National Grid's

'Gone Green' (GG) scenario of future system development. In recognition of the fact that this is a methodological research publication rather than a statement of actual GB adequacy risk levels or wind capacity values for future years, the installed capacity at each wind farm site is slightly adjusted from the actual 2013 GG scenario; however the observations regarding dependence of capacity values on input data are fully representative of the real GB system. The dataset will thus be referred to as 'Adjusted Gone Green' (AGG).

C. Conventional Plant Data

The distribution of available conventional capacity is constructed from individual unit capacities and availability probabilities assuming independence between availabilities of different units. The list of units is that from National Grid's Gone Green scenario, and the unit capacities are again slightly adjusted from those in the GG scenario (for the same reason as wind). The availability probabilities for each class of units are the central estimates from the Great Britain Electricity Capacity Assessment Report [17].

The distribution of available conventional capacity is then constructed by assuming that each plant is either fully available or not available at all, and that the availabilities of the different units are statistically independent. The convolution of these distributions is usually referred to in power systems as a capacity outage probability table [9].

D. Choice of Years of Data

In order to provide a representative range of GB example results, calculations will be performed using the 2013/14 AGG installed capacity and ACS peak demand scenario (a relatively low adequacy risk scenario for GB), and the 2015/16 installed capacity and ACS peak demand scenario (a relatively high risk scenario for GB).

The 2013/14 scenario has ACS peak demand of 55.55 GW, 10.12 GW installed wind capacity, and a distribution of available conventional capacity with mean 58.82 GW and SD 1.95 GW. The 2015/16 scenario has ACS peak demand of 54.59 GW, 12.40 GW installed wind capacity, and a distribution of available conventional capacity with mean 55.91 GW and SD 1.87 GW.

V. RESULTS

A. Relative Shifts of Distributions of Demand and Supply

This section demonstrates the result of Section III-B, namely that if the distribution of $M = X - D$ has a left tail which is lighter than exponential, then a shift in the distribution of M so as to increase risk will result in an increased capacity value of Y .

1) *Distribution Estimation:* For this example

- the distribution of X is as described in Section IV;
- the distribution of D is the empirical distribution of [historic demand rescaled to AGG ACS peak] for data from the 7 historic winters 2005-12;
- the distribution of Y is the empirical distribution of [historic MERRA wind records combined with AGG

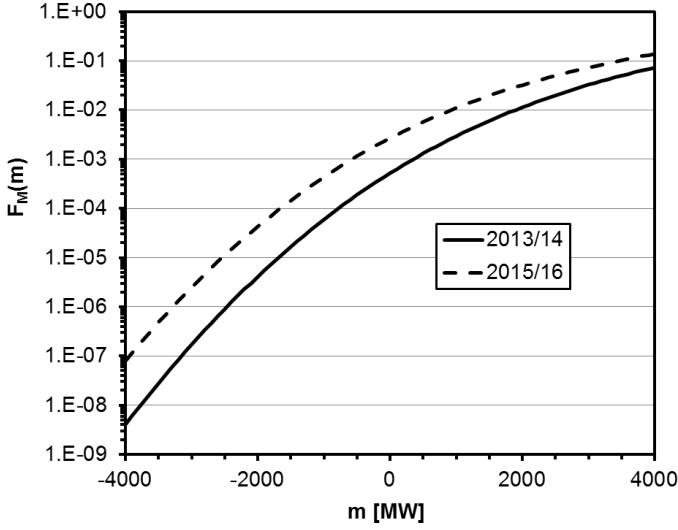


Fig. 1. Snapshot LOLP as a function of required margin for the 2013/14 and 2015/16 AGG scenarios.

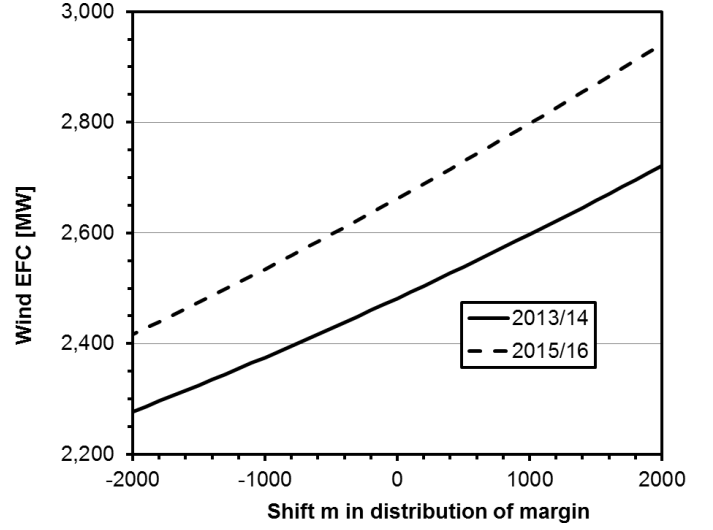


Fig. 2. EFC as a function of margin shift for the 2013/14 and 2015/16 AGG scenarios.

installed capacity scenario] for wind data from the 33 winters 1979-2012.

For estimating the peak season distribution of demand, a 20 week peak (winter) season is used, beginning on the first Sunday in November. The distribution of available wind is estimated from historic wind records from November to March inclusive. M and D are assumed independent.

2) *Effect on LOLP of Shift of Distribution of M* : The cumulative distribution function $F_M(m)$ is plotted in Fig. 1 for the 2013/14 and 2015/16 scenarios. This may be interpreted as the LOLP at a random snapshot in time if the distribution of demand is shifted by m relative to the original scenario; snapshot LOLPs may be converted to LOLE by multiplying by 3360, the number of hours in a 20 week winter.

As required for the result of III-B to hold, the distribution of M is everywhere light-tailed (i.e. decays faster than exponentially) for the examples considered; this follows from inspection of $F_M(m)$ as viewed on a log scale in Fig. 1.

3) *Effect on EFC of Shift of Distribution of M* : Fig. 2 shows the dependence on the distribution shift m of the calculated EFCs for the 2013/14 and 2015/16 AGG scenarios. As predicted in Section III-B, due to the form of $F_M(m)$ the EFC does indeed increase as the distribution of margin is shifted so as to increase the risk level. This commonly observed phenomenon is entirely due to the form of the distribution of M ; if its tail were heavier than exponential then the capacity value would decrease.

B. Upper Limit on Capacity Values

This section demonstrates the result of Section III-A, namely that if there is a probability p that Y lies below a certain level, this places an upper limit on the calculated capacity value of Y , irrespective of its distribution elsewhere.

1) *Data Used*: This demonstration uses the same distribution of M as in Section V-A. The additional generation Y

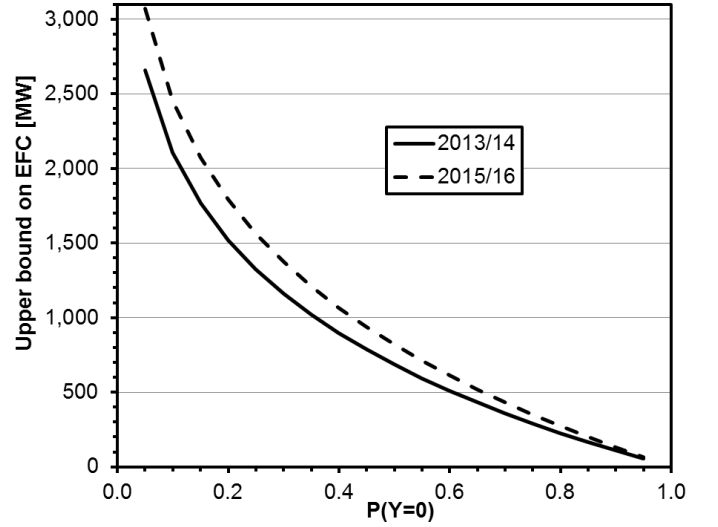


Fig. 3. Upper limit on EFC as a function of the probability of zero available capacity for the 2013/14 and 2015/16 AGG scenarios.

considered has probability p of zero available capacity; it is not necessary to specify other features of its distribution.

2) *Results*: Fig. 3 shows the upper limit on EFC as a function of the probability $p = P(Y = 0)$, for the 2013/14 and 2015/16 AGG scenarios. Section III-A has already noted that this result implies that a probability of near-zero available capacity from Y cannot be fully compensated by the possibility of very high available capacity. It is particularly striking that at high probabilities of zero available capacity, this upper limit can drop to a very low level indeed, to 500 MW or less.

3) *Application: Tidal Barrage*: This provides the clearest demonstration of the reason behind the result in [19] that the capacity value of a single large tidal barrage can be very small as a percentage of installed capacity (the specific example used was a proposed Severn Barrage scheme in SW England, with over 8 GW installed capacity.) This is because typical ebb-

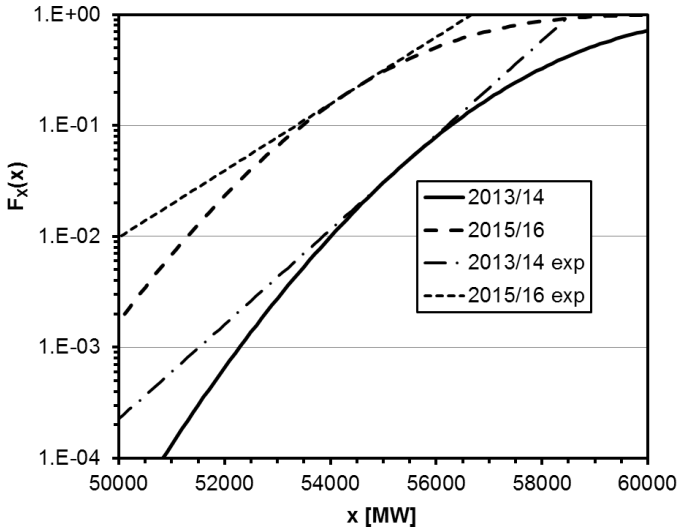


Fig. 4. Exponential approximations to distribution of X for the 13/14 and 15/16 scenarios.

generation schemes only generate when the sea level is low relative to the water stored behind the barrage, i.e. they can generate for less than half of a tidal cycle. For this tidal barrage case, where for researchers outside the project team itself there is inevitably limited access to information on proposed operational practices, this limiting result is particularly valuable as it provides a rigorous upper bound which is independent of the precise detail of the scheme. Specifically, it provides a general result that the contribution of a single large barrage scheme in isolation is very limited; it can only make a substantial contribution to adequacy if there are other facilities whose operation is out of tidal phase with that barrage.

C. Exponential X Approximation

This section demonstrates application of the closed form result in Section III-C for the case where the distribution of X may be approximated by an exponential function in the relevant part of its left tail.

1) *Distribution Estimation for GB Example:* For this GB example, the distribution of X is constructed for a given scenario as described above. A hindcast estimate is used for the joint distribution of Y and D , in which case the EFC is given by the expression in Section III-C3; within this expression, the sum over t is over the joint time series (d_t, y_t) for the seven historic winters 2005-12. The decay constant λ_X is set so that the exponential approximation is tangent to $F_X(x)$ at $x = \text{ACS peak demand}$, see Fig. 4. The hindcast approach is commonly used in applied studies [1], and thus provides a very relevant demonstration of this approximation.

2) *GB Results and Discussion:* The ‘Exponential X ’ EFCs for the 2013/14 and 15/16 scenarios are respectively 1631 MW and 2497 MW; this compares to 1685 MW and 2257 MW for the full hindcast calculation. The errors in the exponential approximation calculation are respectively 3% and 15% of the full hindcast EFC. However, we observe that for this data $F_X(x)$ is not particularly well approximated by

an exponential function over the whole relevant region, and thus even the seemingly good approximation for 13/14 is not very reliable. This closed form result might however be a better approximation in other systems, and the parametrisation of the distribution of X which it provides may be useful in investigating sensitivity of capacity value results to input parameters.

VI. CONCLUSIONS

This paper has presented a number of new theoretical results regarding the capacity value of additional generation, along with examples of how they may be used to explain practical calculation results. We hope soon to explore how the resulting insights may be used to explain the results of capacity value calculations from a range of systems, and differences between these results.

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